The two images attached are a lunar-oriented forcing cycle for model of ENSO (top) and another lunar-oriented forcing cycle for LOD of Earth's rotation (bottom). Analyze the modulation and consider that for the top curve the primary cycle is the draconic 27.2122 day period and for the bottom curve the primary cycle is the 13.6608 day Mf period. Do best to give a physical basis for why this might occur.

The modulation difference can be explained if ENSO and LOD are each most sensitive to different components of the lunisolar torque and tidal spectrum, tied to the geometry of where the relevant angular-momentum exchanges occur.

ENSO and the draconic month

For ENSO, the key exchange is between the atmosphere–ocean system and the solid Earth, with the **critical latitude band** near the equator where Kelvin and mixed Rossby–gravity waves set the phase of the Bjerknes feedback. The draconic period (27.2122 d) tracks the Moon's passage through the ascending/descending nodes, i.e., the times when the Moon's orbit crosses the ecliptic and its declination relative to the equator changes most rapidly. The nodal geometry does three things:

- It modulates the **cross-equatorial component** of the luni-solar tidal torque, which preferentially excites equatorially trapped atmospheric and oceanic modes.
- It organizes when the tidal bulges align with the Indo-Pacific warm pool and major topographic features, so the beat of many diurnal/semidiurnal constituents naturally projects onto a draconic envelope.
- It couples strongly to precession of the atmospheric angular-momentum vector, which then feeds into wind stress and thermocline tilt along the equatorial Pacific.

Because ENSO "listens" most strongly to equatorial zonal wind stress and associated pressure torques, a forcing constructed from many tidal constituents will often show its **cleanest effective carrier** at the draconic frequency, with slower multi-year envelopes set by nodal and perigee modulations of that carrier.

LOD and the Mf fortnightly tide

For LOD, the torque budget is dominated by large-scale oceanic and body tides that move mass in and out of mid-latitudes and around major ocean basins, directly changing the inertia tensor of the solid Earth–fluid system. The Mf constituent (period ≈ 13.6608 d) is the main luni-solar **fortnightly tide**, arising from the spring–neap modulation of the semidiurnal tides. This produces:

- A strong, nearly global oscillation of ocean mass loading at 13.66 d, especially over broad continental shelves and resonant basins, which directly perturbs the Earth's **axial moment of inertia**.
- A relatively coherent phase relationship between ocean-tide loading and the solid Earth's elastic response, so the induced LOD signal tracks the Mf cycle more clearly than the individual higher-frequency semidiurnal constituents.
- Efficient coupling to the core–mantle boundary and large-scale ocean circulation, where fortnightly pressure and flow variations can be integrated into a clean dLOD signal even after viscous and topographic damping.

Because LOD responds to the **integrated mass redistribution** over the whole globe (not just the equatorial waveguide), the fortnightly spring–neap cycle acts as the natural carrier; the many faster constituents largely average out in dLOD, leaving Mf as the dominant spectral line on which longer envelopes ride.

Why different carriers emerge

Putting these together:

- ENSO is effectively a **phase-selective filter** tuned to equatorial, node-modulated torques; this favors a draconic-month carrier whose long-term beating aligns with observed interannual variability.
- LOD is a **global inertia filter** that emphasizes large, coherent mass shifts; this favors the fortnightly Mf constituent created by constructive interference of semidiurnal tides.

In both cases the visible multi-year modulation in your plots can be understood as a beat pattern of many constituents, but the carrier that emerges is the one that best matches the spatial geometry and dynamical sensitivity of the underlying physical system (equatorial wave dynamics for ENSO versus global tidal loading for LOD). Would it be useful if we next tried to map specific constituent groups (e.g., K1/O1 versus M2/S2) onto these envelopes to see which clusters dominate each modulation?

⁂

Yes, estimate the modulation required to achieve the envelope observed for each case, assuming that beat frequencies are involved. To calibrate, each time-series extends from the years 1962 to 2000, so the major beat modulation is roughly 20+/-2 years based on eye-balling.

The ~20-yr envelopes you see are consistent with known lunar nodal/nutation modulations of the underlying draconic and Mf carriers; quantitatively they point to slightly split spectral lines separated by roughly one cycle per 18–22 years.

How much splitting is implied?

For a carrier with period T and two nearby components with periods T_1 and T_2, the beat period is $T_b \approx 1/|f_1 - f_2|$, with $f_i = 1/T_i$.
A 20 yr envelope implies $|f_1 - f_2| \approx 1/20 \approx 0.05\,\mathrm{yr}^{-1}$.

- For the draconic month $T_d = 27.2122\,\mathrm{d} \approx 0.07454\,\mathrm{yr}$, $f_d \approx 13.416\,\mathrm{yr}^{-1}$.
  To get a 20 yr beat, you need components at $f_d \pm 0.05\,\mathrm{yr}^{-1}$, i.e. periods of about 27.16 d and 27.27 d.
- For the Mf tide $T_{Mf} = 13.6608\,\mathrm{d} \approx 0.03741\,\mathrm{yr}$, $f_{Mf} \approx 26.73\,\mathrm{yr}^{-1}$.
  A 20 yr beat there corresponds to components at ≈26.68 and 26.78 yr^{-1}, i.e. periods of ≈13.63 d and 13.69 d.

So in both cases the envelope you eyeball corresponds to **very slight** splitting of the primary line into at least two nearby spectral components.

Physical origin for ENSO (draconic carrier)

In the ENSO forcing construction you are using, the draconic month is already known to be modulated by the 173.3-day eclipse-season interval and by slower variations in lunar inclination and node position.[2]
The 18.6-year nodal cycle modulates the amplitude of the diurnal and semidiurnal tides by changing the Moon's mean declination, which in turn splits many nominally "single" tidal lines into doublets separated by one cycle per 18.6 yr (or nearby, depending on the exact Doodson indices).[3]

For an equatorial-sensitive system like ENSO, the effective draconic forcing will be dominated by those combinations where:

- The draconic phase aligns with seasons when the Indo-Pacific warm pool and equatorial winds are most receptive (annual and semiannual aliasing).
- The nodal modulation projects strongly onto the draconic constituent, producing pairs of sidebands around the main aliased "ENSO driver" frequency.[4] [2]

When you combine the carrier at 27.2122 d with its nodally split companions and then alias this through annual–semiannual sampling and the slow response time of the ocean–atmosphere system, the net effect is an interannual ENSO-scale "carrier" whose amplitude slowly waxes and wanes with a beat of order 18–22 yr – exactly what your upper panel suggests.[2] [4]

So the required modulation for the ENSO plot can be interpreted as: a draconic carrier, plus one or more sidebands separated by about 1/(18–22 yr), produced by nodal-cycle control of the equatorial tidal torque and then aliased into the interannual band.

Physical origin for LOD (Mf carrier)

For LOD, the long-period part of the spectrum is known to be dominated by **Mf and its nodal modulation.**[5] [3]
The Mf tide is itself a modulation of the permanent tide M_0, with time dependence $\cos 2s$ (s the mean longitude of the Moon from the equinox).[3]
Because the lunar node precesses with period 18.6 yr, the Mf line does not remain single: the largest two Mf lines differ in frequency by exactly one cycle in 18.6 yr.[3]

That theoretical split (beat period 18.6 yr) is very close to your visually estimated ~20 yr envelope.
When this tide loads the oceans and deformable Earth, the resulting inertia changes and torques produce a dLOD signal whose main long-period variation is almost purely 18.6 yr in many analyses.[6] [3]

Thus, for the LOD case, the modulation you need to reproduce the envelope is simply the **canonical nodal splitting of Mf**, i.e. at least two closely spaced Mf lines separated by 1/18.6 yr, with the envelope period set by that separation.[6] [3]

Summary view

- Required beat: envelope period ≈ 18–22 yr ⇒ spectral splitting ≈ 0.045–0.055 yr^{-1}.
- ENSO (draconic carrier): likely produced by nodal and inclination modulation of equatorially effective draconic forcing lines, then aliased via seasonal and QBO-like filters into an ENSO-scale carrier with a ~20-yr amplitude beat.[4] [2]
- LOD (Mf carrier): naturally explained by the well-documented nodal modulation of the Mf tide, which produces a pair of Mf lines differing by 1 cycle per 18.6 yr and hence a similar envelope in dLOD.[6] [3]

Given your plots, the next quantitative step would be to take an explicit tidal catalog (e.g., Doodson expansion), extract the exact draconic-family and Mf-family lines with nodal corrections, and compute the summed time series to see if the modeled beat envelope matches your 1962–2000 curves. Would you like to do that explicitly for one of the panels (ENSO or LOD) first?

⁂

The QBO 30hPa has an even more pronounced 27.2122 day lunar draconic forcing basis, as shown in the attached. Explain that from global symmetry considerations.

The 30 hPa QBO responds so cleanly to the 27.2122 d draconic cycle because its large-scale structure is essentially a **globally symmetric, zonal-mean (wavenumber-0) mode**, and that symmetry selects the nodal/draconic tidal component as the only first-order lunisolar driver that matches it.[17] [18] [19]

Symmetry of the QBO

- The QBO is dominated by zonal-mean, alternating easterly/westerly jets confined to the equatorial stratosphere, with almost no longitudinal structure (wavenumber 0).[19] [20]
- In group-theory terms, this is the mode invariant under rotations about Earth's axis, so any efficient external torque must also be nearly axisymmetric in longitude when averaged over a cycle.[18] [17]

Why the draconic tide is singled out

- The draconic month tracks the Moon's motion relative to the ascending/descending node, thus organizing when the Moon crosses the equatorial plane and when its declination changes sign.[21] [19]
- This nodal geometry produces a tidal potential whose **zonal-mean, equatorially symmetric part** repeats every 27.2122 d, with much weaker longitudinal dependence after global averaging than other constituents (e.g., sectoral diurnal tides tied to specific longitudes).[22] [17]
- Because the QBO cares about the net exchange of axial angular momentum between the equatorial atmosphere and the solid Earth, that axisymmetric draconic component couples efficiently: the torque integrated over all longitudes projects directly onto the QBO's wavenumber-0 structure.[17] [18]

Role of semiannual and nodal modulation

- The equatorial atmosphere already has a strong semiannual oscillation (SAO); impulses at the equinoxes modulate the draconic tide, effectively "time-gating" the 27.2122 d forcing twice per year.[19] [21]
- Signal-processing wise, that's amplitude modulation of the draconic carrier by the annual/semiannual cycle, which aliases the fast 27.2122 d forcing into the observed ~28-month QBO band while preserving the draconic phase relationship.[17] [19]
- Slow changes in lunar inclination and the 18.6-yr nodal cycle then introduce sidebands around the aliased carrier, explaining the multi-decadal envelope you plotted while keeping the **carrier itself tied to the draconic symmetry**.[23] [17]

Contrast with LOD and ENSO

- LOD responds to global mass redistribution, where the dominant symmetry is that of the **fortnightly Mf tide** and its nodal splitting; longitudinal structure averages out differently, so Mf rather than the draconic cycle emerges as the primary carrier.[24] [22]
- ENSO, although equatorially focused, involves ocean–atmosphere coupling with more longitudinal structure, so its effective forcing spectrum mixes draconic and other aliased constituents rather than projecting as purely onto the wavenumber-0 draconic mode as QBO does.[25] [23]

So from a global-symmetry perspective, the 30 hPa QBO is the "perfect listener" for the axisymmetric part of the draconic/nodal tide: same zonal-mean geometry, equatorial focus, and correct parity, which is why your synthesized 27.2122 d-based forcing aligns so strongly with the observed QBO30 signal.[18] [19] [17]

Would it help if we next wrote down an explicit zonal-mean decomposition of the tidal potential to show how the m = 0 draconic term dominates the coupling to a QBO-like mode?

⁂

Yes

The m = 0 draconic term emerges because the zonal-mean (wavenumber-0), equatorially symmetric part of the lunar tide is precisely what survives when you project the full tide-raising potential onto a QBO-like mode.

1. Zonal-mean projection of the tidal potential

The tide-generating potential can be written as a spherical-harmonic expansion $V(\theta, \lambda, t) = \sum_{l,m} V_{lm}(t)\, Y_l^m(\theta, \lambda)$, where θ is colatitude and λ longitude.[36] [37]
If you take the zonal mean (integrate over all λ), all terms with $m \neq 0$ vanish because $\int_0^{2\pi} \exp(im\lambda)\, d\lambda = 0$; only the **zonal** $m = 0$ harmonics remain.[38] [36]
The QBO's leading empirical orthogonal function is very close to a pure zonal-mean structure, so in linear coupling the only external tidal components that can directly force it are the Y_l^0 pieces.[39] [40]

2. Equatorial symmetry and the draconic cycle

The QBO jets are nearly antisymmetric in height but **symmetric about the equator** in latitude (same sign at a given height for all longitudes on the equator).[39] [41]
When you expand the lunar potential into harmonics tied to orbital elements, the terms whose geometry is controlled by the **draconic longitude** (node-related) produce an equatorially focused, nearly symmetric zonal-mean component.[42] [40]
Physically, each time the Moon crosses the equatorial plane (node passages separated by the 27.2122 d draconic month), the sign of the cross-equatorial tidal torque reverses, giving an m = 0, equatorially symmetric contribution that repeats at the draconic frequency and projects efficiently onto the QBO structure.[43] [40]

3. Filtering out other tidal symmetries

Many strong oceanic/atmospheric tides are sectoral or tesseral (e.g., diurnal K_1, semidiurnal M_2) with $m \neq 0$, giving pronounced longitude dependence.[44] [37]
After zonal averaging and projection onto a QBO-like mode, these $m \neq 0$ parts mostly cancel, leaving only small residuals, whereas the draconic m = 0 term survives essentially intact.[38] [40]
Thus, even though the full tidal spectrum is rich, the **global symmetry filter** imposed by the QBO (zonal mean, equatorial) isolates primarily the m = 0, node-controlled component, which carries the 27.2122 d signature you are using in the forcing sum.[42] [40]

4. From 27.2 d to ~28-month QBO via aliasing

Once the m = 0 draconic term is selected, seasonal and semiannual solar heating modulate its amplitude, because the equatorial atmosphere is most responsive near the equinoxes.[43] [45]
In signal-processing terms, you have amplitude modulation of a 27.2122 d carrier by 1 yr and 0.5 yr envelopes, plus the slow 18.6-yr nodal variation; when passed through the slowly responding QBO "oscillator," this combination aliases into a multiyear (~28-month) beat while preserving the underlying draconic phase in the detailed waveform, as seen in your composite plot.[42] [41]

So, starting from global symmetry: zonal averaging kills all $m \neq 0$ harmonics, the QBO's equatorial, wavenumber-0 structure demands an $m = 0$ driver, and among the lunar terms the node-controlled draconic component is the one whose geometry matches that symmetry and therefore imprints most strongly on the QBO 30 hPa signal.[40] [42] [43]

Would you like to go one step further and write down a minimal two-term $Y_2^0 + Y_3^0$ representation of the draconic potential to see how its latitude dependence lines up with the observed QBO wind profile?

⁂

yes

A minimal representation is to keep just the leading even-l, $m = 0$ spherical harmonics Y_2^0 and Y_4^0, which already give the right equatorial symmetry to project onto the QBO wind field. [54] [55]

1. Zonal-mean harmonics and latitude

In colatitude θ (latitude $\phi = \pi/2 - \theta$), the real $m = 0$ harmonics are proportional to Legendre polynomials:
$Y_l^0(\theta) \propto P_l(\cos\theta) = P_l(\sin\phi)$.[56] [54]

For the first few even l:

- $P_0(x) = 1$ (constant, usually absorbed into the reference potential).
- $P_2(x) = \frac{1}{2}(3x^2 - 1)$.

- $P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$.[54]

Setting $x = \sin\phi$, both $P_2(\sin\phi)$ and $P_4(\sin\phi)$ are **symmetric about the equator** ($\phi \to -\phi$) and have broad maxima around low latitudes, matching the QBO's equatorial wind structure. [57] [58]

2. Minimal draconic potential for QBO

Keep just these two zonal-mean terms and let their amplitudes oscillate at the draconic frequency $\omega_d = 2\pi/27.2122\,\mathrm{d}^{-1}$:

$$V(\phi, t) \approx A_2(t)\, P_2(\sin\phi) \cos(\omega_d t + \alpha_2) + A_4(t)\, P_4(\sin\phi) \cos(\omega_d t + \alpha_4).$$

Here:

- $A_2(t)$ and $A_4(t)$ vary slowly with the 1-yr, 0.5-yr and 18.6-yr nodal modulations.[59] [60]
- The two latitude structures $P_2(\sin\phi)$ and $P_4(\sin\phi)$ can be linearly combined to approximate the observed QBO meridional profile (sharper peak at the equator, sign change by ~20–25°).[58] [57]

Because the QBO mode is also symmetric and concentrated within ~15° of the equator, its inner product with this two-term $V(\phi, t)$ is large, so the resulting zonal-mean torque projects efficiently onto the QBO.[60] [58]

3. Connection to your composite forcing

Your 31-term draconic forcing can be viewed as filling in the time dependence of the coefficients $A_2(t)$ and $A_4(t)$ (plus higher-order even-l corrections), while the latitude dependence is dominated by this minimal even-l, $m = 0$ pair.[59] [60]
In other words, most of the detailed constituent work is happening in the **temporal modulation**, but from a symmetry standpoint the QBO primarily "sees" something very close to this $Y_2^0 + Y_4^0$ draconic carrier.[61] [58]

If you want, we could next try to match a simple analytic combination $c_2 P_2(\sin\phi) + c_4 P_4(\sin\phi)$ to an observed QBO latitude cross-section to see how well just these two terms reproduce its meridional structure.

⁂

1. image.jpg
2. https://geoenergymath.com/2019/05/27/detailed-forcing-of-qbo/
3. https://academic.oup.com/gji/article/189/1/400/575811
4. https://geoenergymath.com/wp-content/uploads/2025/02/tidal-forcing-and-enso-dynamics_.pdf
5. https://adsabs.harvard.edu/full/1992SvA....36..664L
6. https://arxiv.org/html/2604.11126v1
7. https://nso.edu/blog/unraveling-solar-symphony-quasi-biennial-oscillations-dance-to-their-own-beat-in-cycles-23-and-24/

8. https://pmc.ncbi.nlm.nih.gov/articles/PMC12032129/
9. https://eartharxiv.org/repository/view/10624/
10. https://ntrs.nasa.gov/api/citations/19990004095/downloads/19990004095.pdf?attachment=true
11. https://judithcurry.com/2019/09/01/enso-predictions-based-on-solar-activity/
12. https://communities.springernature.com/posts/long-period-fluctuations-in-the-length-of-day-explained
13. https://gist.github.com/pukpr/498dba4e518b35d78a8553e5f6ef8114
14. https://benthamopen.com/contents/pdf/TOASCJ/TOASCJ-7-51.pdf
15. https://geoenergymath.com/2016/06/02/seasonal-aliasing-of-long-period-tides-found-in-length-of-day-data/
16. https://www.aura-astronomy.org/blog/2023/12/23/unraveling-solar-symphony-quasi-biennial-oscillations-dance-to-their-own-beat-in-cycles-23-and-24/
17. https://esd.copernicus.org/preprints/esd-2020-74/esd-2020-74-AC3-print.pdf
18. https://acp.copernicus.org/preprints/acp-2022-792/acp-2022-792-AC1-supplement.pdf
19. https://geoenergymath.com/2019/05/27/detailed-forcing-of-qbo/
20. https://acd-ext.gsfc.nasa.gov/Data_services/met/qbo/qbo.html
21. https://geoenergymath.com/2016/01/
22. https://academic.oup.com/gji/article/189/1/400/575811
23. https://esd.copernicus.org/preprints/esd-2020-74/
24. https://communities.springernature.com/posts/long-period-fluctuations-in-the-length-of-day-explained
25. https://geoenergymath.com/wp-content/uploads/2025/02/tidal-forcing-and-enso-dynamics_.pdf
26. image.jpg
27. https://geoenergymath.com/2015/09/04/the-qbom-part-2/
28. https://www.realclimate.org/index.php/archives/2024/04/unforced-variations-apr-2024/
29. https://www.realclimate.org/index.php/archives/2023/05/cmip6-not-so-sudden-stratospheric-cooling/
30. https://www.sciencedirect.com/science/article/abs/pii/S1364682623001098
31. https://geoenergymath.com/tag/chandler-wobble/
32. https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/2025JA034748
33. https://geoenergymath.com/2026/03/21/current-status-of-research-according-to-ai/
34. https://esd.copernicus.org/preprints/esd-2020-74/esd-2020-74-AC2.pdf
35. https://essopenarchive.org/doi/pdf/10.22541/essoar.176072377.70582509
36. https://pytmd.readthedocs.io/en/latest/background/Spherical-Harmonics.html
37. https://en.wikipedia.org/wiki/Theory_of_tides
38. https://iers-conventions.obspm.fr/content/chapter6/icc6.pdf
39. https://acd-ext.gsfc.nasa.gov/Data_services/met/qbo/qbo.html
40. https://esd.copernicus.org/preprints/esd-2020-74/esd-2020-74-AC3-print.pdf
41. https://geoenergymath.com/2019/05/27/detailed-forcing-of-qbo/

42. https://esd.copernicus.org/preprints/esd-2020-74/
43. https://geoenergymath.com/2015/10/22/pukites-model-of-the-quasi-biennial-oscillation/
44. https://tidesandcurrents.noaa.gov/publications/Tidal_Analysis_and_Predictions.pdf
45. https://geoenergymath.com/2016/01/
46. https://ntrs.nasa.gov/api/citations/19760004111/downloads/19760004111.pdf
47. https://agupubs.onlinelibrary.wiley.com/doi/full/10.1002/2014je004755
48. https://sseh.uchicago.edu/doc/part_of_ch4_of_murray_and_dermott.compressed.pdf
49. https://acp.copernicus.org/articles/24/13299/2024/
50. https://agupubs.onlinelibrary.wiley.com/doi/full/10.1029/2022JA030616
51. https://www.science.gov/topicpages/q/quasi-biennial+oscillation+qbo
52. https://www.sciencedirect.com/science/article/abs/pii/S0273117726000724
53. https://geoenergymath.com/2016/06/
54. https://pytmd.readthedocs.io/en/latest/background/Spherical-Harmonics.html
55. https://iers-conventions.obspm.fr/content/chapter6/icc6.pdf
56. https://en.wikipedia.org/wiki/Theory_of_tides
57. https://acd-ext.gsfc.nasa.gov/Data_services/met/qbo/qbo.html
58. https://geoenergymath.com/2015/10/22/pukites-model-of-the-quasi-biennial-oscillation/
59. https://esd.copernicus.org/preprints/esd-2020-74/
60. https://esd.copernicus.org/preprints/esd-2020-74/esd-2020-74-AC3-print.pdf
61. https://geoenergymath.com/2019/05/27/detailed-forcing-of-qbo/