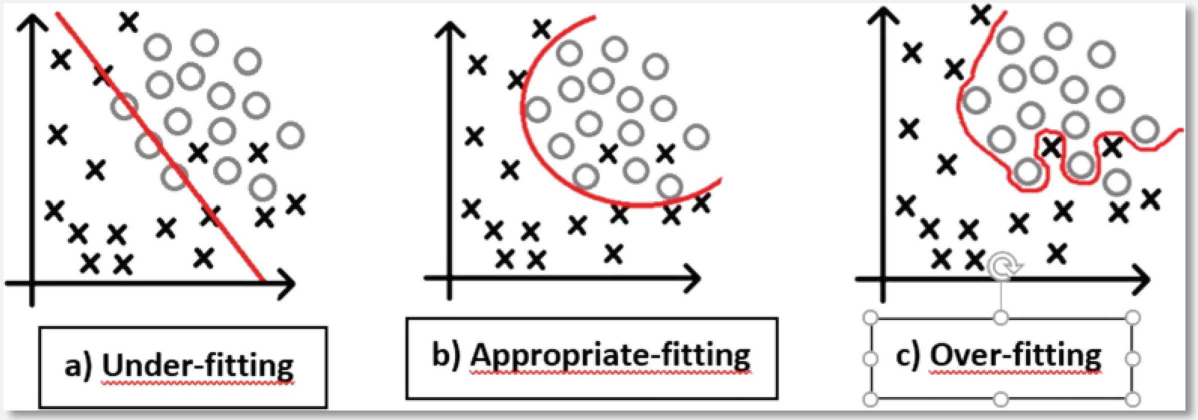
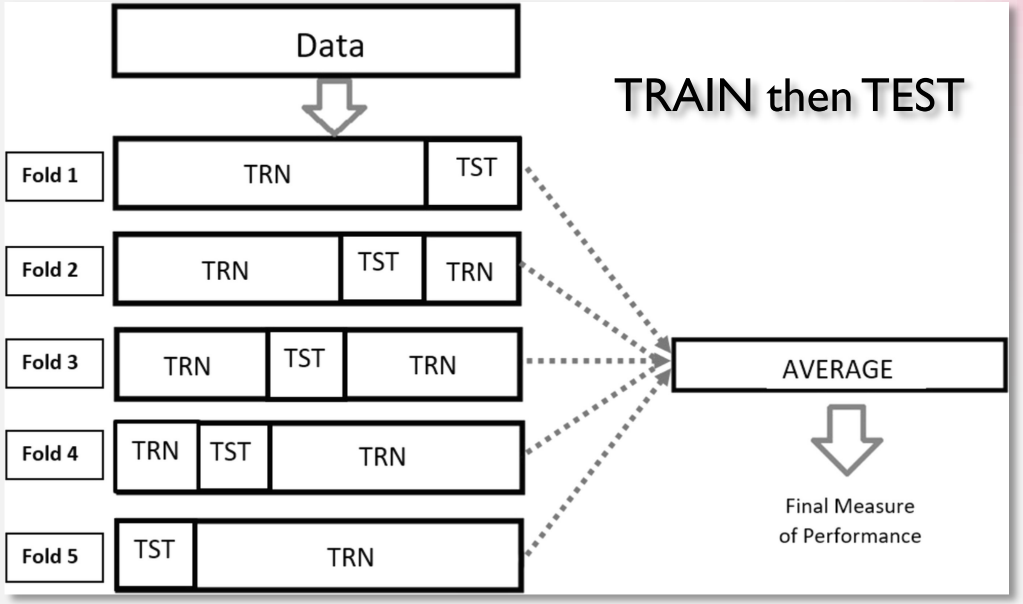
ENSO Modeling

Cross-Validation





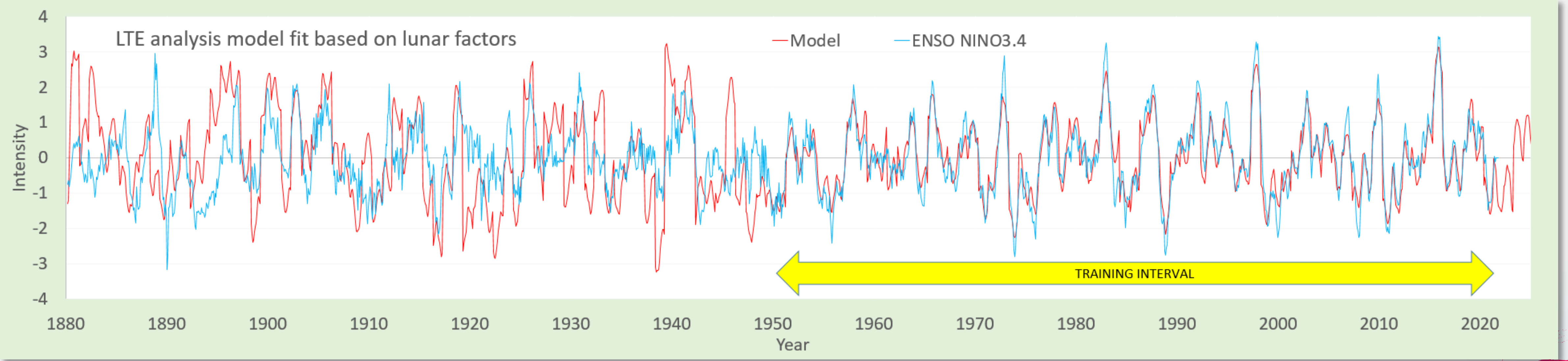
Cross-Validation primer

Montesinos López O.A., Montesinos López A., Crossa J. (2022) *Overfitting, Model Tuning, and Evaluation of Prediction Performance.* In: Multivariate Statistical Machine Learning Methods for Genomic Prediction. Springer, Cham. https://doi.org/10.1007/978-3-030-89010-0_4

Appropriate Fit of ENSO

Using Laplace's Tidal Equation model - see *Mathematical Geoenergy* (Wiley, 2018)

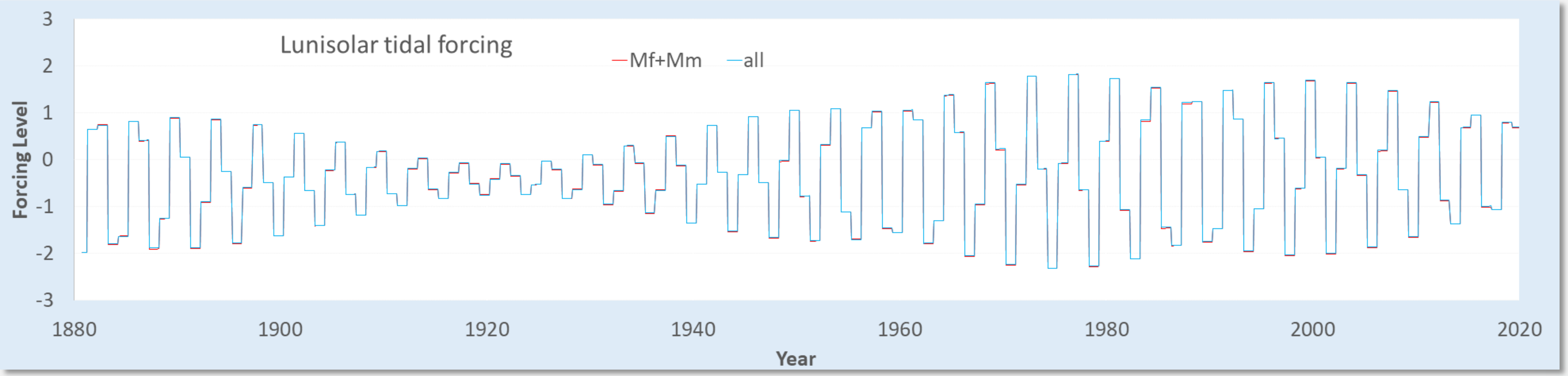
- This is a parsimonious fit as it applies 2 primary tidal factors, **Mf** (13.66 day)+**Mm** (27.55 day)
- An over-fit training interval reproduces back-fitted values, capturing most El Nino events



Paul Pukite

Tidal Forcing

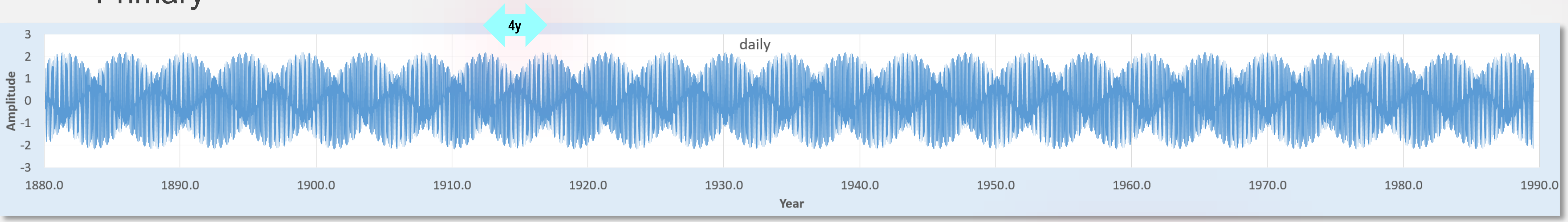
- Input forcing is an annual impulse modulated by the tidal amplitude at that time
- The **Mf** and **Mm** amplitudes are supplemented by other known tidal amplitudes
- As shown below the effect of adding all factors is slight



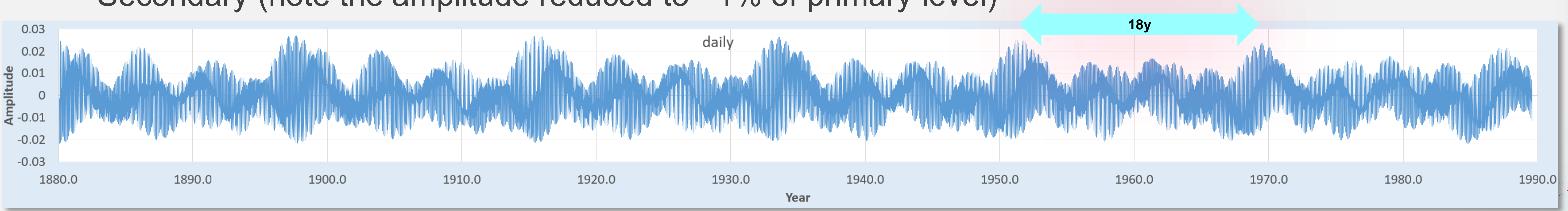
Paul Pukite

Tidal Factor Breakdown

- At the daily level, the **Mf+Mm** factors produce the well-known 4.42 year perigean envelope
- The secondary factors combine to produce an 18 year Saros cycle and 6 year sub-cycle
  - Primary



  - Secondary (note the amplitude reduced to ~1% of primary level)



Paul Pukite

Laplace's Tidal Equations derivation

Starting point

- The primitive equations

12.2.5. Part 1: Deriving a Closed-Form Solution to Laplace's Tidal Equations

For a fluid sheet of average thickness D, the vertical tidal elevation ζ, and the horizontal velocity components u and v (in the latitude φ and longitude λ directions), the following is the set of Laplace's tidal equations. The idea is that along the equator, that is, for φ at zero, we can reduce these to something much simpler:

$$\frac{\partial\zeta}{\partial t}+\frac{1}{a\cos(\varphi)}\left[\frac{\partial}{\partial\lambda}(uD)+\frac{\partial}{\partial\varphi}(vD\cos(\varphi))\right]=0,$$
$$\frac{\partial u}{\partial t}-v(2\Omega\sin(\varphi))+\frac{1}{a\cos(\varphi)}\frac{\partial}{\partial\lambda}(g\zeta+U)=0, \qquad (12.1)$$
$$\frac{\partial v}{\partial t}+u(2\Omega\sin(\varphi))+\frac{1}{a}\frac{\partial}{\partial\varphi}(g\zeta+U)=0,$$

where Ω is the angular frequency of the planet's rotation, g is the planet's gravitational acceleration at the mean ocean surface, a is the planetary radius, and U is the external gravitational tidal forcing potential.

Ending point

- After applying ansatz (see Chap 12)

After properly applying the chain rule, this reduces the equation to a function of $\zeta(t)$ and $\varphi(t)$, along with a constant A. The A subsumes the wavenumber SW(s) portion, so there will be multiple solutions for the various standing waves, which will be used in fitting the model to the data:

$$A\zeta(t)+\frac{1}{\frac{\partial\varphi}{\partial t}}\cdot\frac{\partial}{\partial t}\frac{\zeta'(t)}{\frac{\partial\varphi}{\partial t}}=0 \qquad (12.12)$$

So, if we fix $\varphi(t)$ to a periodic function with a long-term mean of zero

$$\frac{\partial\varphi}{\partial t}=\sum_{i=1}^{i=N}k_i\omega_i\cos(\omega_i t) \qquad (12.13)$$

to describe the perturbed tractive latitudinal displacement terms near the equator, then the solution is the following potentially highly nonlinear result (depending on the strength of the inner terms):

$$\zeta(t)=\sin\left(\sqrt{A}\sum_{i=1}^{i=N}k_i\sin(\omega_i t)+\theta_0\right) \qquad (12.14)$$

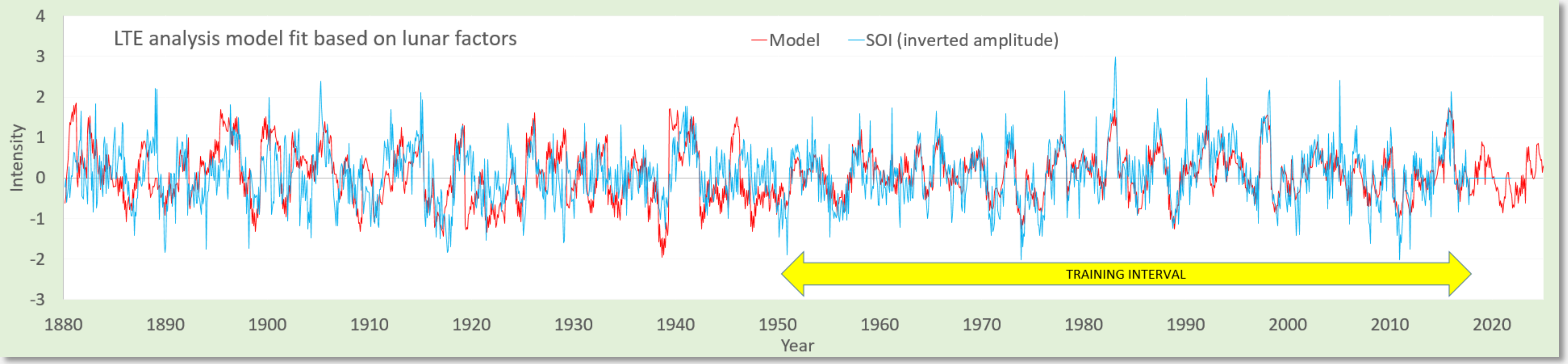
where A is an aggregate of the constants of the differential equation and θ_0 represents the fixed phase offset necessary for aligning on a seasonal peak. This approximation of a

Fit standing wave modes

Paul Pukite

Higher-Order terms?

- SOI duplicates NINO34 but has finer structure
- If these are higher index standing wave modes then it may be possible to fit as well

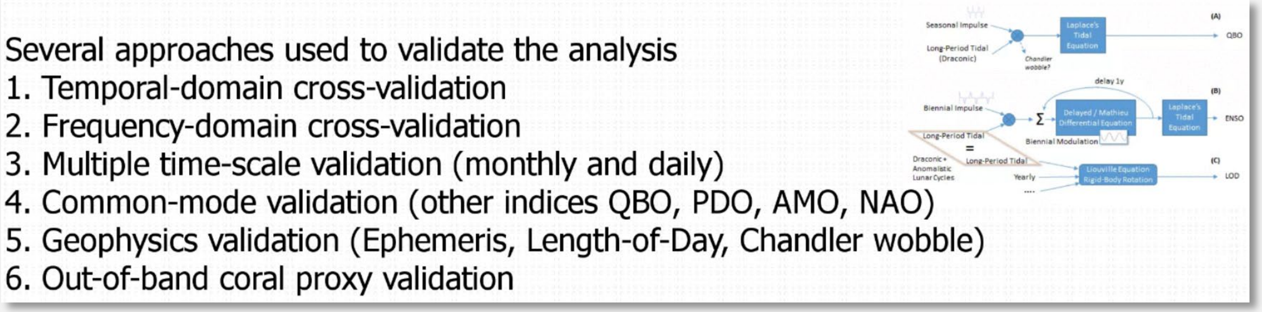


Paul Pukite

Path Forward

- ENSO model is simple and parsimonious, but nonlinear terms make it challenging to fit
- Invitation is open to apply other cross-validation approaches such as described here:

Ephemeris calibration of Laplace's tidal equation model for ENSO AGU 2018 Fall Meeting



- Pointless to have to wait years to validate true forecasts, so this is what we must test, test, test

Paul Pukite