Supplementary Presentation for
"Nonlinear long-period tidal forcing with application to ENSO, QBO, and Chandler wobble"

Comparing conventional tidal analysis and length-of-day models to Chandler wobble, QBO, and ENSO

#1 Tidal. Performing tidal analysis involves fitting a model to a historical sea-level height (SLH) tidal gauge time-series. The amplitudes and phases derive from known fundamental lunisolar sinusoidal tidal cycles and higher frequency harmonics (all figures are also provided as separate files, and data files and source code from https://github.com/pukpr/GeoEnergyMath).

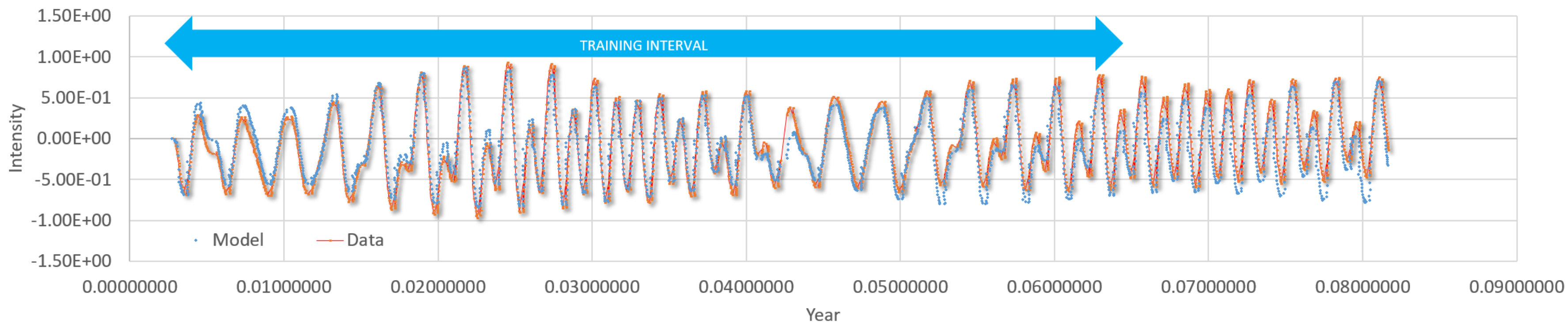

__Figure 1:__ Conventional tidal analysis by fitting amplitudes and phases of known tidal periods

Data: from Mathematical Geoenergy (Wiley/2018), p. 180 https://github.com/pukpr/GeoEnergyMath/blob/master/dartb.txt

#2 LOD. A similar tidal harmonics fitting approach applies for the differential length-of-day (dLOD) time-series, as it involves straightforward additive ***long-period*** tidal cycles, which are at least several days longer than the primarily diurnal and semi-diurnal cycles of conventional tidal analysis.

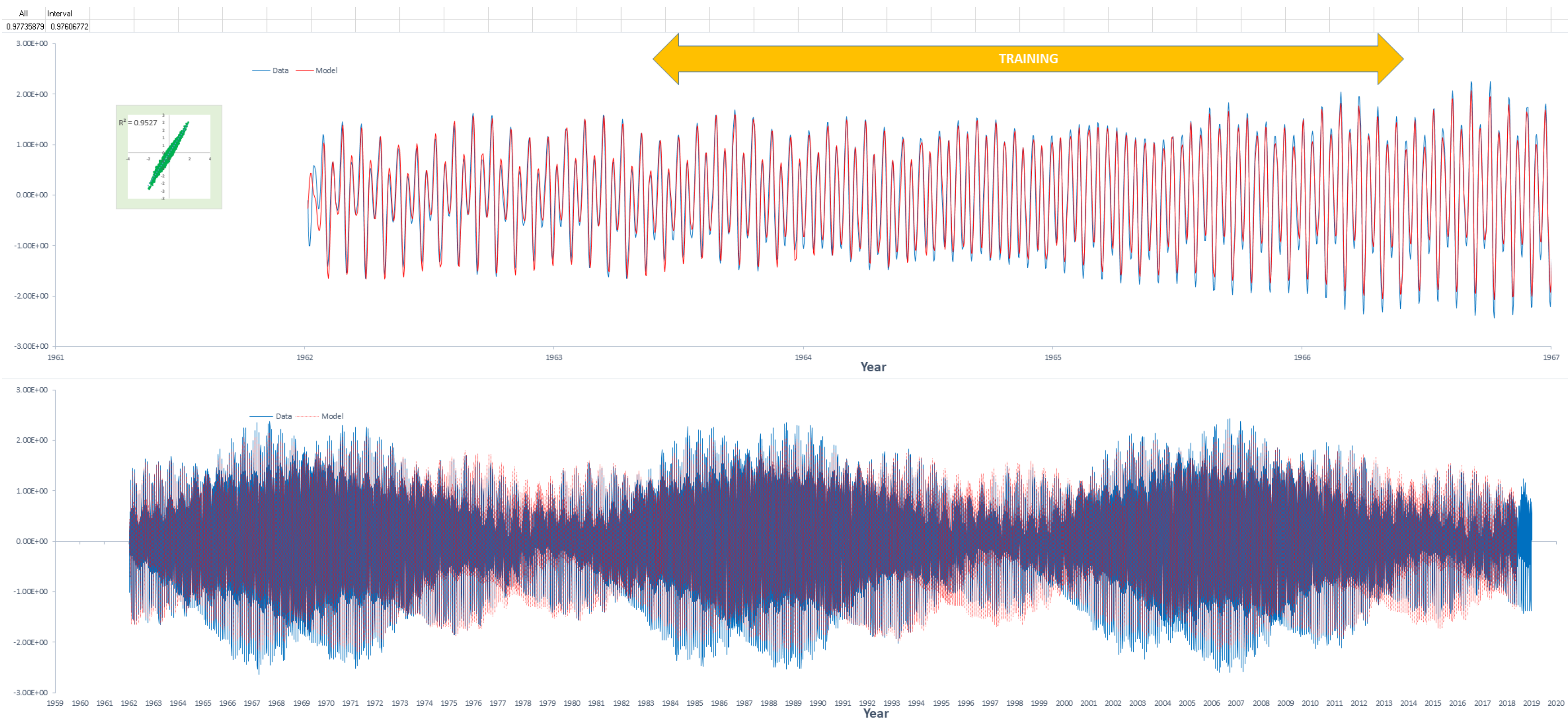

Figure 2: *Long-period tidal cycles map directly to dLOD. The long-term modulation shown follows the 18.6 year nodal cycle*

Data: from International Earth Rotation and Reference Systems Service http://hpiers.obspm.fr/eoppc/eop/eopc04/eopc04.62-now

#3 Chandler wobble. The Chandler wobble model assumes a single monthly tidal forcing and multiplies it by a semi-annual nodal impulse (one for each nodal cycle pass). This is slightly more complex than **#1** or **#2**. This forced response approach extends from the previous two examples, and is a plausible alternative to a stochastic forcing stimulating a resonance.

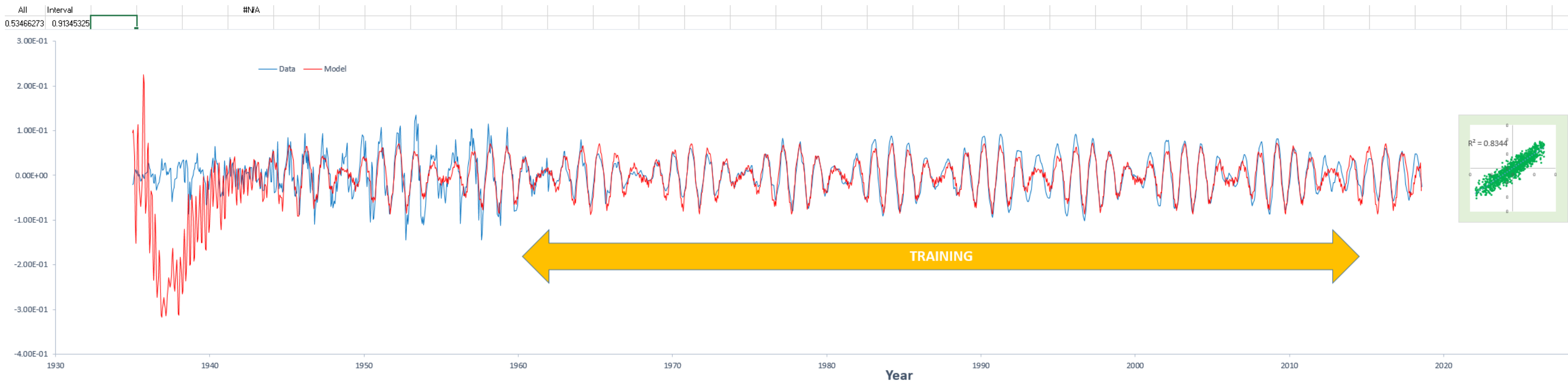

Figure 3: *Chandler Wobble model consisting of a lunisolar tidal forcing. The initial stimulus natural response quickly damps out, revealing the forced response. The beat frequency is annual (sun) nodal against lunar nodal cycle.*

Data: from International Earth Rotation and Reference Systems Service https://datacenter.iers.org/data/latestVersion/38_EOP_C01.1900-NOW_V2013_0138.txt

#4 QBO. The QBO model is impulse-modulated by nearly the same mechanism as for the Chandler wobble of **#3**. But instead of a bandpass response filter for the Chandler wobble, the QBO model applies an integrating filter to create more of a square-wave-like time-series. Again, this is a forced response as a plausible alternative to the natural or resonant response that is often proposed to explain QBO.

.

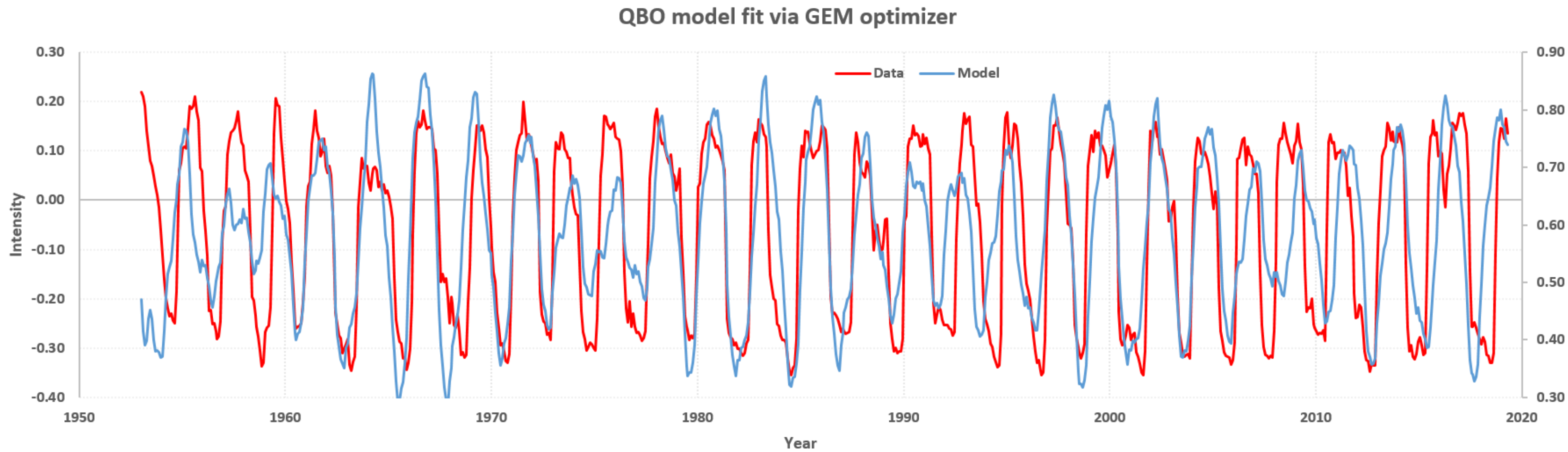

Figure 4*: QBO model of 30 hPa layer has a fundamental cycle of ~2.33 years. This odd cycle is a result of aliasing of the lunar nodal cycle against the annual cycle*

Data: from Free University Berlin https://www.geo.fu-berlin.de/met/ag/strat/produkte/qbo/qbo.dat

#5 ENSO. The ENSO model adds the nonlinear Laplace's Tidal Equation (LTE) modulation to the square-wave-like fit of **#4** (QBO), tempered by being calibrated by the tidal forcing model for **#2** (dLOD). The analysis flow is portrayed in Figure 5 via steps A through G below. The ENSO model is complex due to the non-linearity of the solution. The cyclic tidal factors can create harmonics from both the inverse cubic gravitational pull and from the LTE solution, and together with the annual impulse modulation creates an additional aliasing that requires an exhaustive iterative fitting process to reveal.

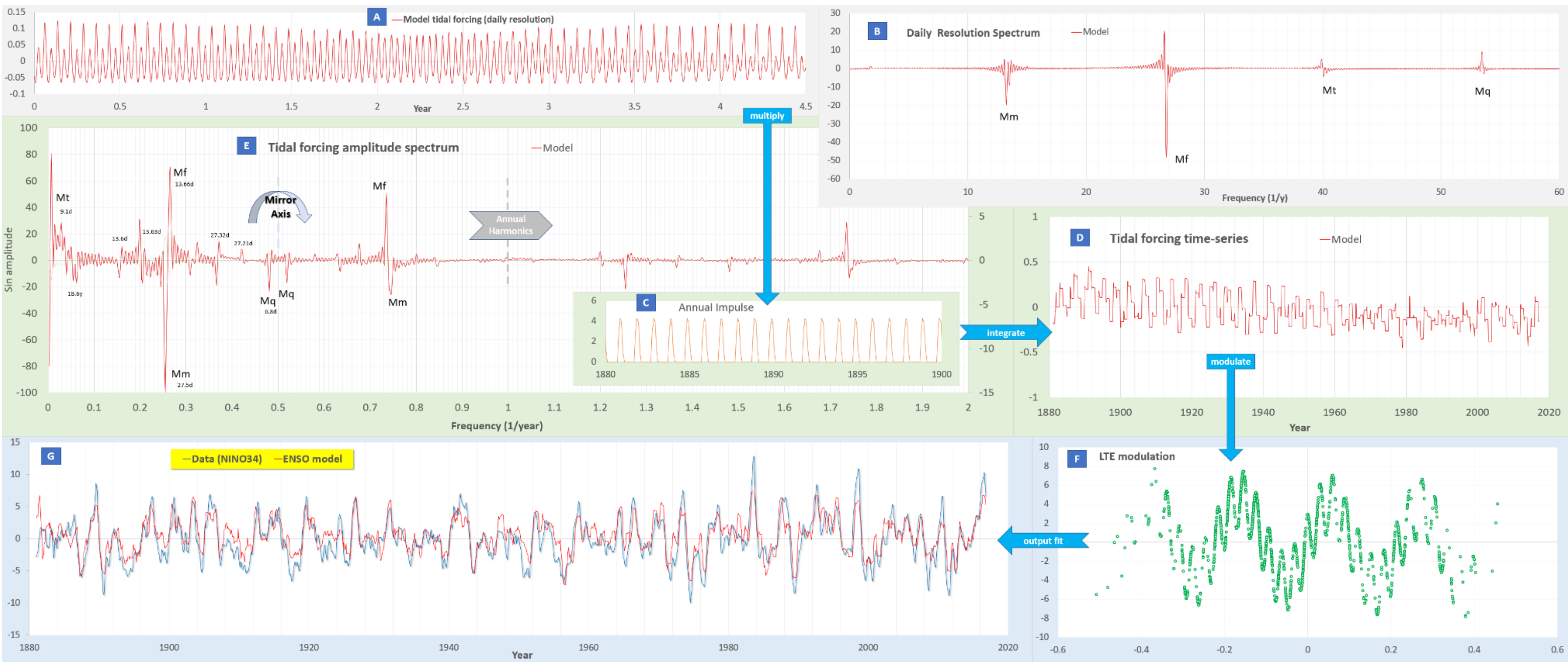

Figure 5: Higher complexity of ENSO (NINO34) model due to nonlinear modulation of the LTE solution

Data: from NOAA Physical Sciences Laboratory https://psl.noaa.gov/gcos_wgsp/Timeseries/Data/nino34.long.data